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WEIGHT ESTIMATES OF FRACTIONAL INTEGRATION OPERATORS

ANNOTATION

Dissertations for the degree of Doctor of Philosophy (PhD) in the specialty 6D060100-Mathematics

The relevance of the topic. The dissertation work is devoted to the conditions and problems of obtaining weight estimates of integral operators of unit order of the Hardy type**.**

In the theory of weight spaces, one of the main problems is the characterization of estimates of integral and discrete operators of fractional order of the Hardy type. The evaluation of these operators began to be considered in connection with the increasing applications in harmonic analysis, the theory of differential, difference equations and other areas of mathematics. For integral and discrete operators of fractional order, their weight estimates are considered in many cases, and the mathematical literature related to them contains many books and scientific papers.

Since the 60s of the last century, weight estimates of the conditions of integral, discrete Hardy-type operators consisting of two weights and two parameters have been studied. In more recent times, people have become interested in estimating the weight of conditions of Hardy-type operators consisting of various ratios of three weights and four parameters, for example, the issues of two-way estimation and obtaining compactness of integral operators of fractional order of Hardy type are considered.

Relevance of the dissertation work:

- limitation and compactness of the fractional integral operator with logarithmic features;
- double estimates and compacts of integral operators of fractional order, the upper and lower limits of which are functions;
- the necessary and sufficient conditions for weight estimates of three weight and four-parameter inequalities are considered.

Integral and discrete Hardy-type inequalities and their various generalized types are studied by many countries around the world. For example, scientists from Russia, Sweden, Czech Republic, USA, Croatia, Italy, Norway, China.

The study of Hardy - type inequalities in Kazakhstan began with the works of M. Otelbaeva and R. Oynarova, which continued in the works of K. Myrzataeva, A. Kalybai, A.M. Temirkhanova, A.M. Abylaeva, M. Aldai. The dissertations of S. H. Shalginbaeva, A.M. Temirkhanova, A.M. Abylaeva were defended on the topic of the dissertation.

The purpose of the work. The main purpose of the dissertation is to obtain criteria for the boundedness and compactness of a fractional-order integral operator of the Hardy type with a logarithmic singularity, as well as to obtain two-way

estimates and compactness of fractional-order operators whose upper and lower limits are functions in Lebesgue weight spaces, with various parameters.

The object of the study. A fractional-order integral operator of the Hardy type with logarithmic and α-singularities in the Lebesgue weight space. In addition, integral operators of fractional order, the upper and lower limits of which are functions, as well as three weight functions and four parametric integral inequalities.

Research methods. Along with the methods of mathematical and functional analysis, the dissertation uses the method of dividing fractional order into integral parts "localization method" (the blocking technique). Various classical inequalities, as well as Hardy-type weight inequalities, were used in the proof process.

Scientific novelty. All the results obtained are new. Among them:

1. Necessary and sufficient conditions for weights under which a fractional order operator of the Hardy type with a logarithmic singularity acts bounded and compactly in Lebesgue weight spaces at $0<\alpha<1$, $\frac{1}{\alpha}<p<q<\infty$ and $\beta\geq 0$;

2. Necessary and sufficient conditions for weights under which a fractional order operator of the Hardy type with a logarithmic singularity acts bounded and compactly in Lebesgue weight spaces at $0<\alpha<1$, $0<q<p<\infty$, $p>\frac{1}{\alpha}$ and $\beta\geq 0$.

3. Criteria for the boundedness and compactness of fractional order operators with upper and lower limits of which are functions in Lebesgue weight spaces when the parameters satisfy conditions $1<p\leq q<\infty$, $\frac{1}{p}<\alpha<1$ and $\beta\geq 0$.

4. Criteria for the boundedness and compactness of fractional order operators with upper and lower limits of which are functions in Lebesgue weight spaces when the parameters satisfy the conditions $0<\alpha<1$, $0<q<p<\infty$, $p>\frac{1}{\alpha}$ and $\beta\leq 0$

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Theoretical and practical value. The work is theoretical in nature. In it, weight estimates for fractional order operators of the Hardy type with logarithmic and singularities are obtained, and weight estimates of fractional order operators are obtained, which upper and lower limits are functions. The obtained results can be applied in the evaluation of solutions of differential equations modeling various processes in biology, quantum physics and technical problems.

Personal contribution of the applicant. The research papers presented in the dissertation were carried out with the direct participation of the author. The criteria for the fulfillment of three weighted, four parametric linear inequalities are considered: the boundedness and compactness of an integral operator of fractional order with a logarithmic singularity, as well as two-sided evaluation and compactness of Hardy-type operators, the upper and lower limits of which are functions. The results were published in the form of scientific articles and scientific reports.

Approbation of the results obtained.

1. At the traditional international scientific April conferences of the Institute of Mathematics and Mathematical Modeling (Almaty 2019, 2020).

2. At the international conference "Theoretical and Applied Problems of Mathematics, Mechanics and Computer Science" (2019).

3. Report at the international conference "Actual Problems of Analysis, Differential Equations and Algebra" dedicated to the 10th anniversary of the Eurasian Mathematical Journal (EMJ-2019) (Nur-Sultan, 2019).

Individual results of the dissertation work:

- At the scientific seminar "Functional analysis and its application" (seminar leaders are academicians of NAS RK M. Otelbayev and R. Oynarov, professors E. D. Nursultan, K. N. Ospanov) made a report 2 times;

- from September 26 to December 26, 2019, during the scientific internship of a doctoral student, under the guidance of a professor of the Department of Mathematics and Statistics of the Faculty of Physics and Mathematics of the Bashkort State Pedagogical University. M. Akmulla, Ufa, Republic of Bashkortostan, Ph.D., Honored Scientist of the Russian Federation and the Republic of Belarus, laureate of the State Prize of the Republic of Belarus in the field of science Sultanaev Yaudat Talgatovich presented reports at scientific seminars "Spectral theory of differential operators" conducted under his leadership.

- Repeatedly reported and discussed at the scientific seminar "Weight inequalities and their application" (seminar leaders academician of NAS RK R. Oynarov, associate professors A.M. Temirkhanova, A.M. Abylaeva, associate Professor M. Aldai).

Publications. 11 papers have been published on the topic of the dissertation, including 1 article in a rating journal indexed in the Scopus database, Web of Science (Web of Science, Impact factor 88%, 2022, Q1), 3 articles in scientific publications included in the list submitted by the Committee for Control in the field of education and Science MES RK, 7 publications in the materials of international scientific conferences, including 1 publication in the materials of foreign international conferences.

The structure and scope of the dissertation. The dissertation work contains a thematic page, content, introduction, two parts, conclusion and a list of references from 58 titles. The total volume of the dissertation is 97 pages.

In the second chapter, necessary and sufficient conditions for the parameters of the integral operator of fractional order are obtained for the following cases:

а) $0<\alpha<1,\ \frac{1}{\alpha}<p<q<\infty$ and $\beta\geq 0$ (section 2.1);

б) $0<\alpha<1,\ 0<q<p<\infty,\ p>\frac{1}{\alpha}$ and $\beta\geq 0$ (section2.2);

c) $0<\alpha<1$, $\frac{1}{\alpha}<p<q<\infty$ and $\beta\geq 0$ (section2.3);

a) $0<\alpha<1$, $\frac{1}{\alpha}<p<q<\infty$ and $\beta\leq 0$ (section 2.4);

б) $0<\alpha<1$, $0<q<p<\infty$, $p>\frac{1}{\alpha}$ and $\beta\leq 0$ (section 2.5);

c) $0<\alpha<1$, $\frac{1}{\alpha}<p<q<\infty$ and $\beta\leq 0$ (section 2.6).

In conclusion, a brief analysis of the results obtained is given and the environment of their application is outlined. The dissertation work is completed with a list of references.